Student Number:

Fort Street
High School

Teacher:

Class:

2022 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Approved calculators may be used
- A reference sheet is provided
- In Questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks : 100

Section I – 10 marks (pages 5 – 11)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 12 – 38)

- Allow about 2 hours and 45 minutes for this section
- This section is divided in to 6 parts
- Write your student number on each part.
- Attempt Questions 11 – 31

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1. What are the values of x for which $f(x)=\sqrt{x^2-3x}$ is defined?

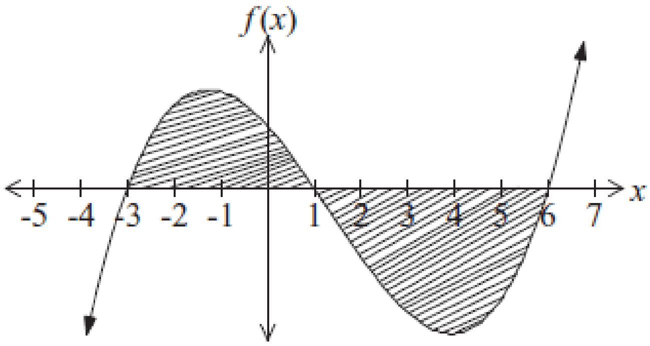
A. $0 \le x \le 3$

B. $x \ge 3$

C. $x \le 0$ or $x \ge 3$

D. *all real x*

2. In which one of the following ways can the shaded area in the diagram be calculated?



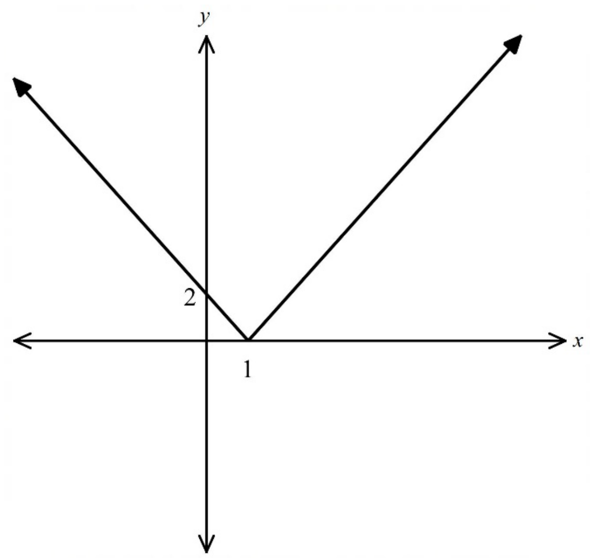
A. 0

B. $\int_{-3}^{1} f(x)dx + \int_{1}^{6} f(x)dx$

C. $2\int_{-3}^{1} f(x)dx$

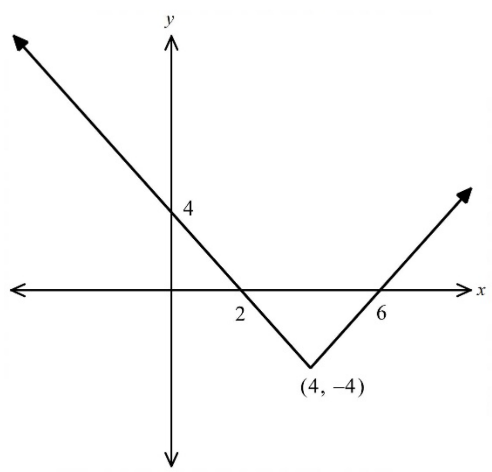
D. $\int_{-3}^{1} f(x)dx - \int_{1}^{6} f(x)dx$

3. The graph of $y = f(x)$ is shown below.



Which of the graphs below represents $y = f(x+3) + 4$?

A.



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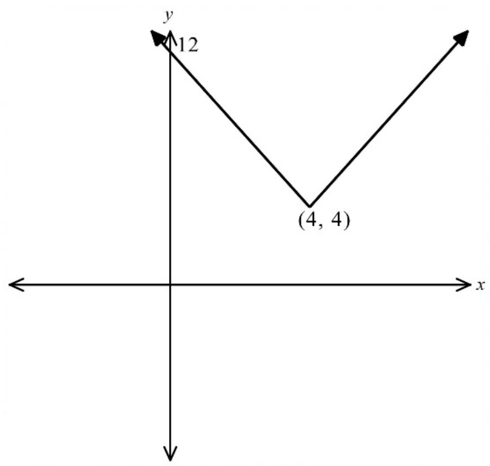

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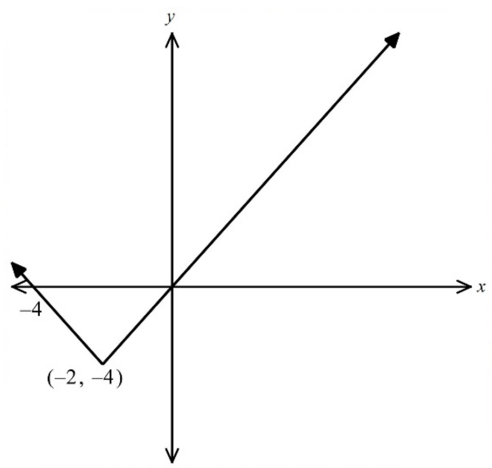
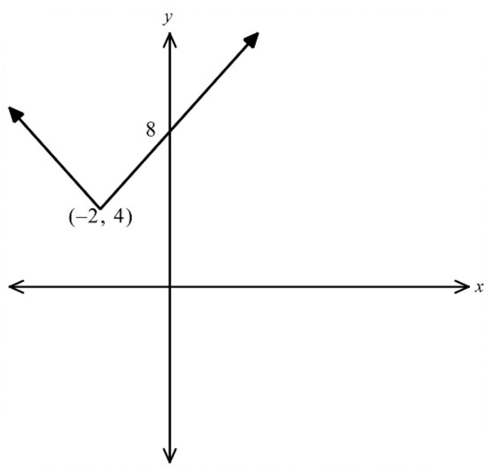
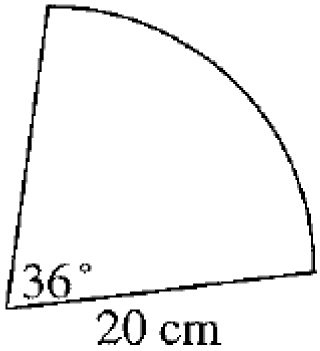


D.



4. What is the perimeter P, of the sector given below with angle $36°$ and radius 20 centimetres?



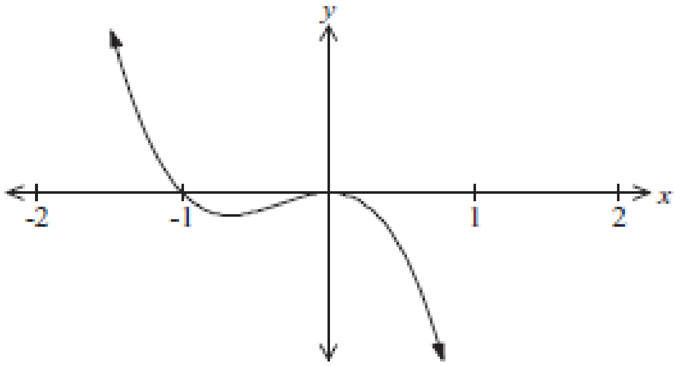
A. $(40+4\pi)\ cm$

B. $\left(0.5\times 400\times \frac{\pi}{5}\right)\ cm$

C. $(40+36^{\circ})\ cm$

D. $0.5\times 400\times\left(\frac{\pi}{5}-\sin\frac{\pi}{5}\right)\ cm$

5. The graph of a function $y = f(x)$ is shown below.



Which graph would represent $y = f'(x)$?

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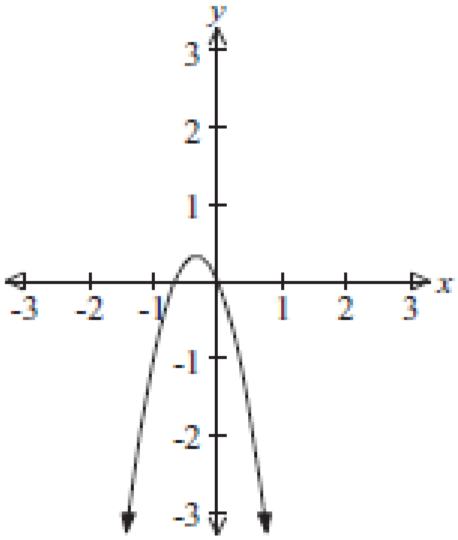
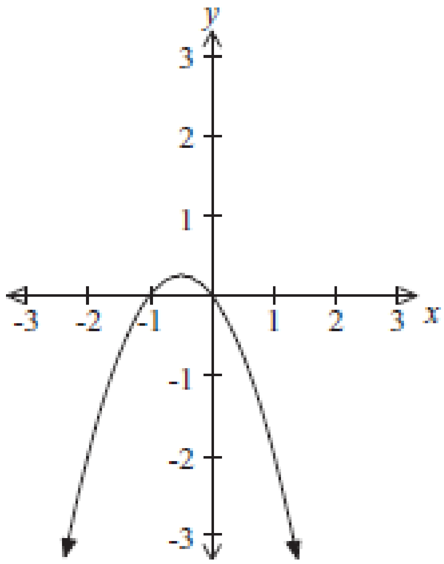
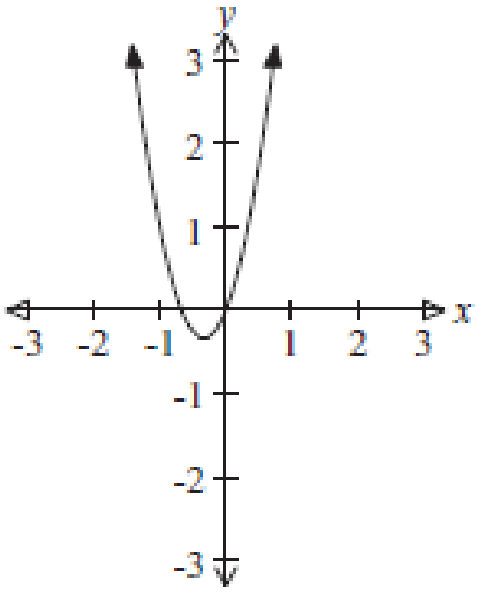


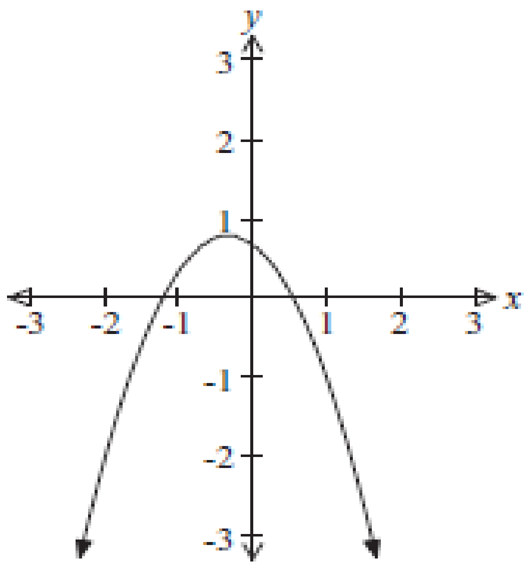
B.



C.



D.



6 The population N ,of a colony of ants grows exponentially according to the formula $N(t) = 550e^{kt}$, where k is the growth constant and t is time in days.

If $N(\ln 4) = 13750$, what is the exact value of k ?

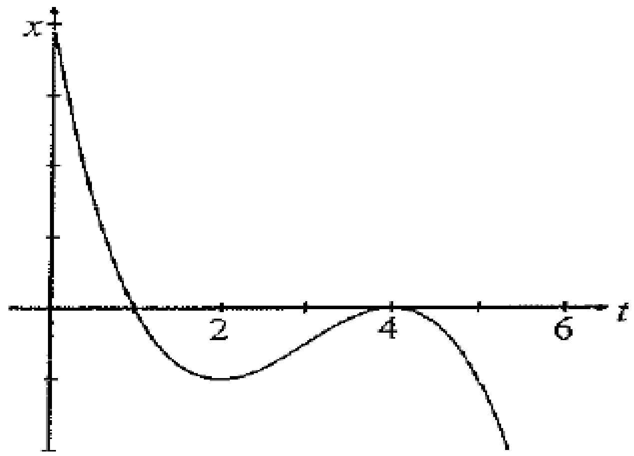
A. $\dfrac{25}{\ln 4}$

B. $\dfrac{\ln 25}{\ln 4}$

C. $\dfrac{\ln 4}{25}$

D. $\ln\left(\dfrac{25}{4}\right)$

7. The displacement, x metres, from the origin of a particle moving in a straight line at any time, t seconds, is shown in the graph below



When was the particle at rest?

A $t = 0$

B. $t = 2$ and $t = 4$

C. $t = 1$ and $t = 4$

D. $t = 1$, $t = 2$ and $t = 4$

8. What is $\frac{d}{d\theta}\left(\frac{2-\sin\theta}{\cos\theta}\right)$?

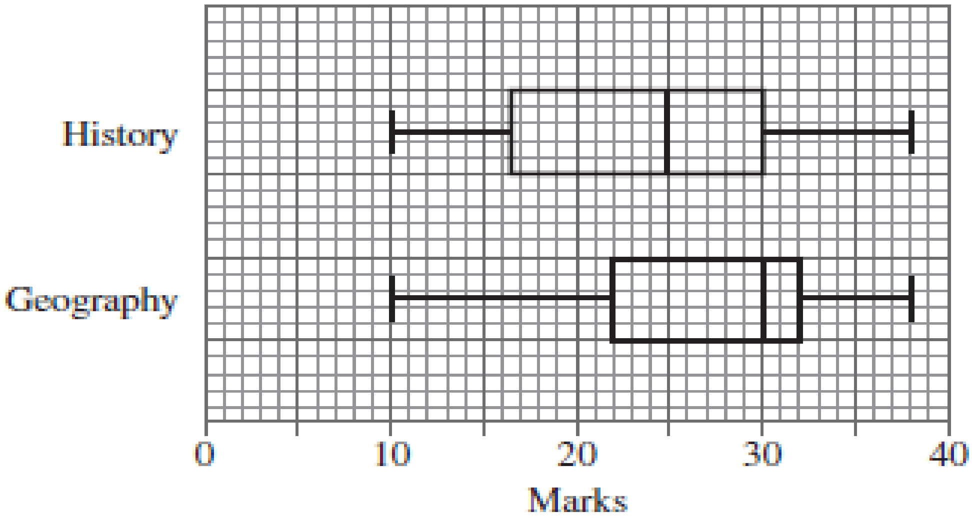
A. $\frac{2\sin\theta - 1}{\cos^2\theta}$

B. $\frac{1-2\sin\theta}{\cos^2\theta}$

C. $\frac{-\cos^2\theta - 2\sin\theta + \sin^2\theta}{\cos^2\theta}$

D. $\frac{2\cos\theta + 2\sin\theta - 1}{\cos^2\theta}$

9. The box and whisker plots given below show the results of a history and a geography test.



In History, 112 students completed the test. The number of students who scored above 30 marks was the same for the History test and the Geography test.

How many students completed the Geography test?

A. 8

B. 50

C. 56

D. 112

10. There are 10 green marbles and W white marbles in a bag. The probability of selecting a white marble is $\frac{4}{9}$. How many more white marbles need to be added to the bag so that the probability of selecting a white marble from the bag is $\frac{3}{5}$?

A. 7

B. 8

C. 14

D. 21

Section II
Part A

Student Number:

Question 11 (2 marks)

Simplify fully: $\dfrac{3}{x+2} \times \dfrac{x^2+2x}{6x-3}$ **2**

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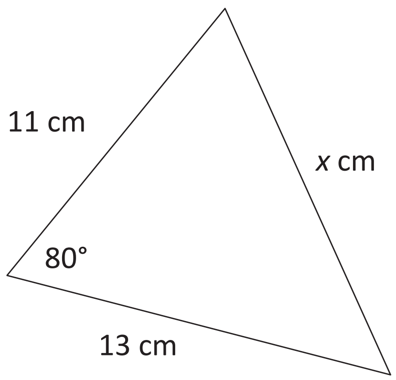
Question 12 (2 marks)

In the following diagram, use the Cosine Rule to find the value of x, correct to two decimal places **2**



NOT TO SCALE

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Question 13 (2 marks)

If $f(x) = (x-5)^2$ and $g(x) = \sqrt{x} \ -5$,find $g\left(f\left(x\right)\right)$ **2**

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Question 14 (3 marks)

The first term of an arithmetic series is 4. The fifth term is four times the third term. Find the sum of the first 10 terms. **3**

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Question 15 (3 marks)

Solve $\log_e(2x+1)-\log_e 3=3$ **3**

PART B

Student Number:

Question 16 (5 marks)

(a) Show that $(\text{cosec}^2\theta - 1)\sin^2\theta = \cos^2\theta$ **2**

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(b) Hence, or otherwise solve $(\text{cosec}^2\theta - 1)\sin^2\theta = \dfrac{3}{4}$ for $-\pi \le x \le \pi$ **3**

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Question 17 (3marks)

(a) Let $f(x)=\begin{cases}1-x^2 & for \quad x\le 1\\ x-2 & for \quad x>1\end{cases}$

Sketch the function for $-2\le x\le 4$ **2**

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(b) What happens to the graph $x=1$? **1**

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Question 18 (4 marks)

(a) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x \, \mathrm{d}x$ giving the answer in exact form with rational denominator. **2**

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(b) Find $\int x^2 \left(x^3 - 1\right)^5 \, dx$ **2**

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Question 19 (6 marks)

A function is given by $f(x) = 3 + 4x^3 - x^4$.

(a) Find the stationary points and determine their nature. **3**

Question 19 (continues)

(b) Show that there is a point of inflexion at $(2,19)$ on the curve. **1**

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(c) Hence, Sketch the graph of the curve and clearly label the stationary points and the points of inflexion on the curve. DO NOT determine the x intercepts of the curve **2**

PART C

Student Number

Question 20 (5 marks)

(a) Show that $\dfrac{1+e^x}{1-e^x} = 1+\dfrac{2e^x}{1-e^x}$ **2**

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(b) Hence, find $\displaystyle\int \frac{1+e^x}{1-e^x}\,dx$ **3**

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Question 21 (3 marks)

A normal to the graph of $y = 2\sqrt{x}$ has the equation $y = -3x + b$. Find the value of b. **3**

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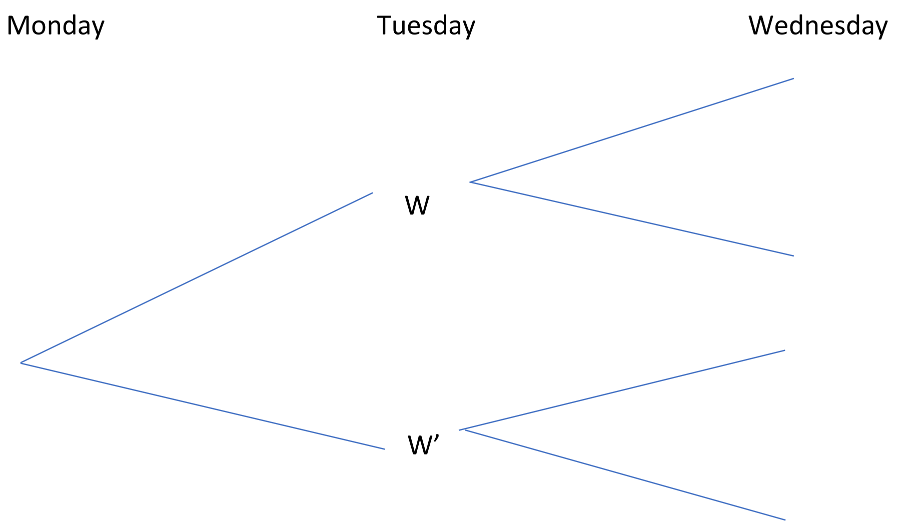
Question 22 (4 marks)

The probability of Sirhind soccer team winning their next game is dependent on whether or not they win the previous game.

If they won the previous game, the probability of winning their next game is 0.8
If they did not win their previous game, then the probability of winning the next game is 0.4

They won on Monday and then they played on Tuesday and Wednesday.

(a) Represent the above information using the tree diagram given below **2**



(b) Find the probability that the Sirhind soccer team do not win on Wednesday. **2**

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Question 23 (3 marks)

For the probability distribution below, find the values of m and n given that the expected value of the distribution is 4.8 **3**

x	0	2	3	7	n
$P(X=x)$	0.1	0.3	m	0.3	0.2

PART D

Student Number

Question 24 (5 marks)

If $f(x)=\log_e\left(x+\sqrt{x}\right)$

(a) Show that $f'\left(x\right)=\dfrac{2\sqrt{x}+1}{2x\left(\sqrt{x}+1\right)}$ **3**

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Question 24 (continued)

(b) Hence, when $x = 2$, Show that $f^{'}(x) = \dfrac{2\sqrt{x}+1}{2x\left(\sqrt{x}+1\right)}$ can be written in the form $a + b\sqrt{2}$ **2**

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Question 25 (9 marks)

Consider the function defined by $f(x)=e^{2x}(1-x)$ where $-3\le x\le 1$

(a) Complete the following table **1**

x	-3	-2	-1	0	1
$f(x)$	0.01	0.05			

(b) Using the trapezoidal rule with five function values, approximate the area under the curve $f(x)$ for $-3\le x\le 1$. **2**

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Question 25 (continued)

(c) Differentiate $f(x) = e^{2x}(1-x)$ and show that the function has only one stationary point. **3**

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Question 25 (continued)

(d) Without further calculus, use the table of values from part (a) to sketch

$f(x) = e^{2x}(1-x)$ for $-3 \le x \le 1$, and show the stationary point on the curve. **2**

(e) From the diagram, decide whether the approximation underestimates or overestimates the true value of the area under the curve. Give a brief reason. **1**

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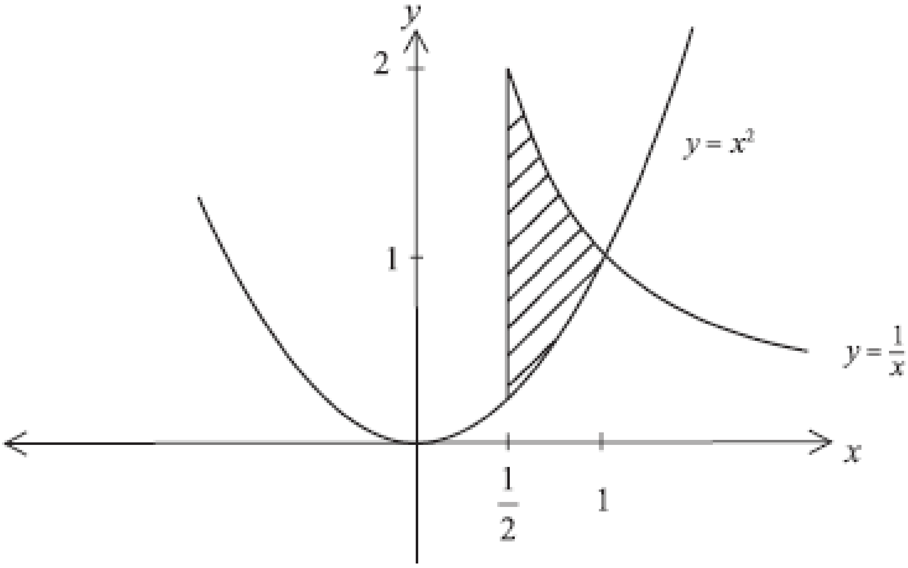
Question 26 (3 marks)

Calculate the exact area of the shaded region in the diagram below enclosed by the curves $y = x^2$, $y = \dfrac{1}{x}$

and the line $x = \dfrac{1}{2}$ **3**



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PART E

Student Number

Question 27 (3 marks)

Consider the function $f(x) = \frac{1}{2} - \frac{1}{2^x + 1}$

(a) Show that $\frac{1}{2} - \frac{1}{2^x + 1} = \frac{2^x - 1}{2(2^x + 1)}$ **1**

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(b) Hence determine whether $f(x)$ is even, odd or neither. Show all working. **2**

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Question 28 (6 marks)

The velocity v of a particle initially at the origin is given by $v(t) = \sin\left(\frac{1}{4}t\right)$, in metres per second.

(a) Find the displacement function $x(t)$ **2**

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(b) Find the acceleration function $a(t)$ 1

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(c) Find the value of x when $t = 4\pi$ 1

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Question 28 (continued)

(d) Sketch the displacement-time graph on the domain $0 \le t \le 16\pi$ and briefly describe the motion. **2**

Question 29 (7 marks)

The rate of fuel burn, t minutes after the engine starts operation, R kg per minute, is given by the relation

$$R = 10 + \frac{10}{1+2t}$$

(a) Draw a sketch of R as a function of t , $t \geq 0$ **2**

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(b) What is the rate of burn, R, after 7 minutes? **1**

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Question 29 (continued)

(c) What value does R approaches as t becomes very large? **1**

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(d) Calculate the total amount of the fuel burnt in the first 7 minutes. **3**

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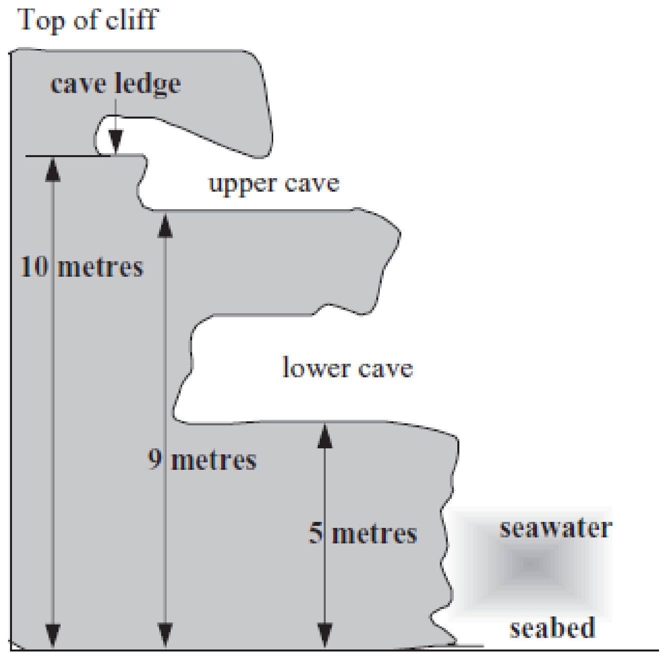
PART F

Student Number

Question 30 (5 marks)

Carol, Mary and Manar arrive at the top of a cliff at 9:00 am on a particular Saturday morning. They are planning to explore the lower cave in the cliff.



The height, h metres of the seawater is given by the function $h(t) = 7 - 4\sin\left(\frac{\pi}{7}t\right)$, where t is the number of hours after 9:00 am, on Saturday morning.

(a) What is the height of the water when the girls first arrive at the top of the cliff? **1**

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(b) The girls notice that the height of the water is falling and they wait for the water to fall so that they can explore the lower cave. When the water is 4 metres and falling, they enter the lower cave. What time do the girls enter the lower cave? **2**

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Question 30 (continued)

(c) The girls plan to leave the lower cave when the water level is again at 4 metres and rising.

Calculate how long will they have to explore the lower cave to the nearest minute? **2**

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Question 31 (7 marks)

A wire of length 5 metres is to be bent to form the hypotenuse and base of a right angled triangle ABC with right angle at B. Let the length of the base BC be x metres.

(a) Write an expression for the length of the hypotenuse AC in terms of x. **1**

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(b) Show that the area of the triangle ABC is $\frac{1}{2}x\sqrt{25-10x}$ square meters. **2**

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Question 31 (continued)

(d) Find the maximum possible area of the triangle ABC . **4**

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End of Paper

Fort Street High School

Solutions

Student Number:

Teacher:

Class:

2022 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Approved calculators may be used
- A reference sheet is provided
- In Questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks : 100

Section I – 10 marks (pages 5 – 11)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 12 – 38)

- Allow about 2 hours and 45 minutes for this section
- This section is divided in to 6 parts
- Write your student number on each part.
- Attempt Questions 11 – 31

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1. What are the values of x for which $f(x)=\sqrt{x^2-3x}$ is defined?

$x^2 - 3x \geq 0$

$x(x-3) \geq 0$

$x \leq 0$ or $x \geq 3$

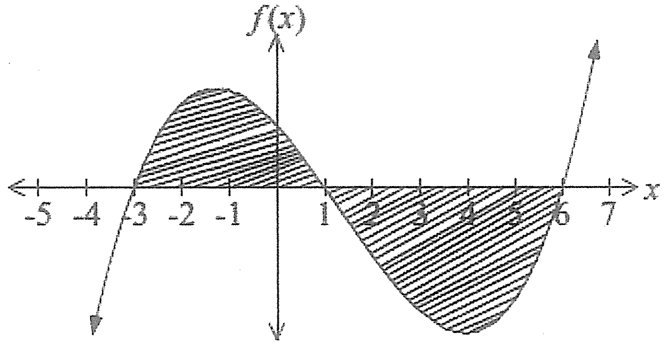
A. $0 \leq x \leq 3$

B. $x \geq 3$

(C.) $x \leq 0$ or $x \geq 3$

D. *all real x*

2. In which one of the following ways can the shaded area in the diagram be calculated?



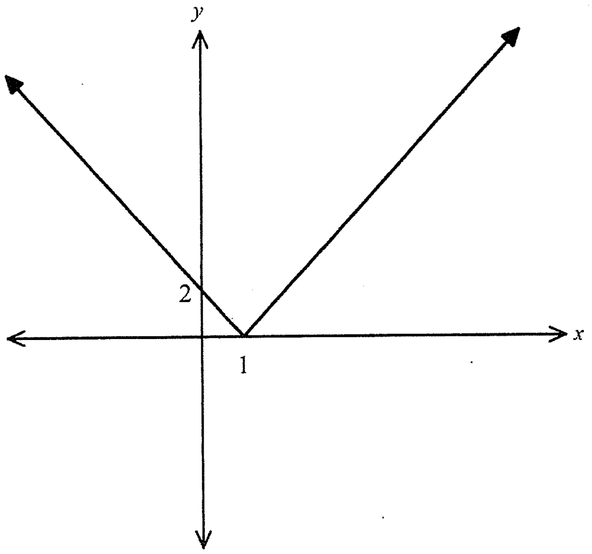
A. 0

B. $\int_{-3}^{1} f(x)dx + \int_{1}^{6} f(x)dx$

C. $2\int_{-3}^{1} f(x)dx$

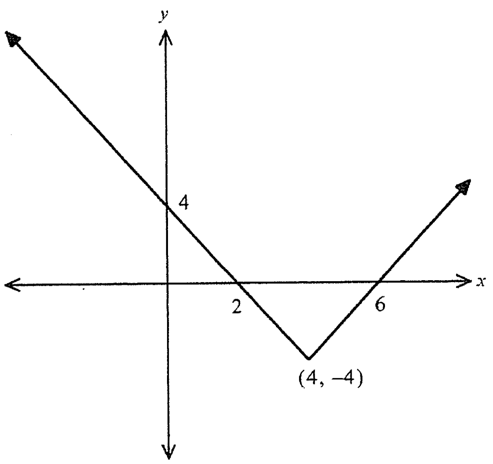
(D.) $\int_{-3}^{1} f(x)dx - \int_{1}^{6} f(x)dx$

3. The graph of $y = f(x)$ is shown below.

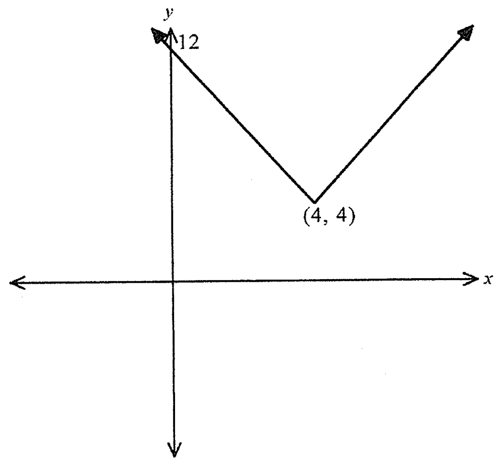


Which of the graphs below represents $y = f(x + 3) + 4$?

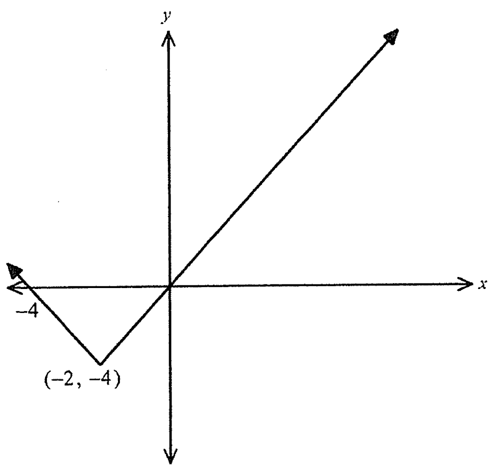
A.



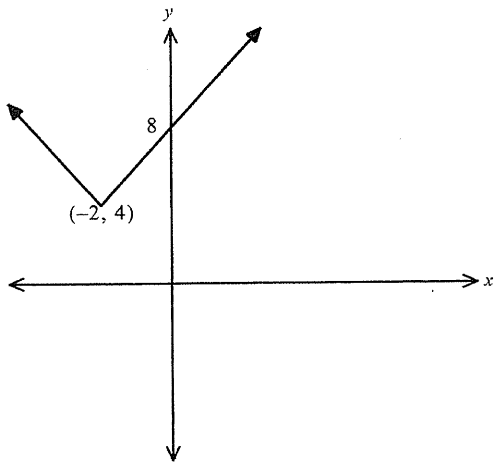
B.



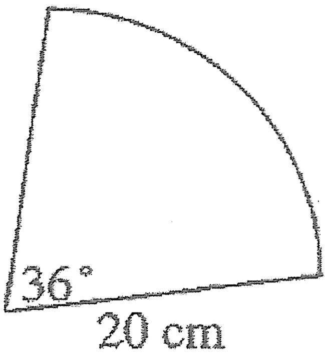
C.



D.



4. What is the perimeter P, of the sector given below with angle 36° and radius 20 centimetres?



$36^\circ = \frac{36}{180} \times \pi$ radians

$= \frac{\pi}{5}$ radians

$l = r\theta = 20 \times \frac{\pi}{5} = 4\pi$

Perimeter $= 20 + 20 + 4\pi$

$= 40 + 4\pi$

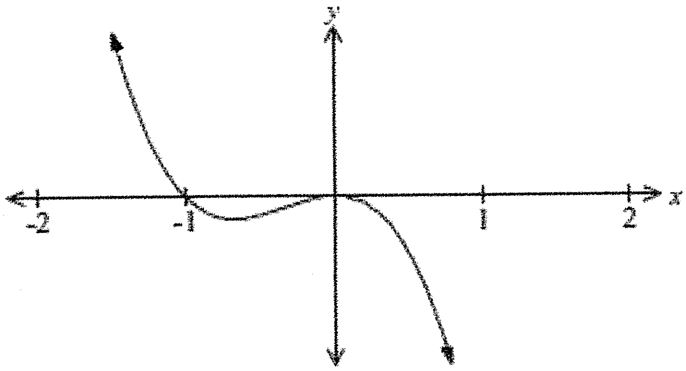
(A.) $(40 + 4\pi)\ cm$

B. $\left(0.5 \times 400 \times \frac{\pi}{5}\right)\ cm$

C. $(40 + 36^\circ)\ cm$

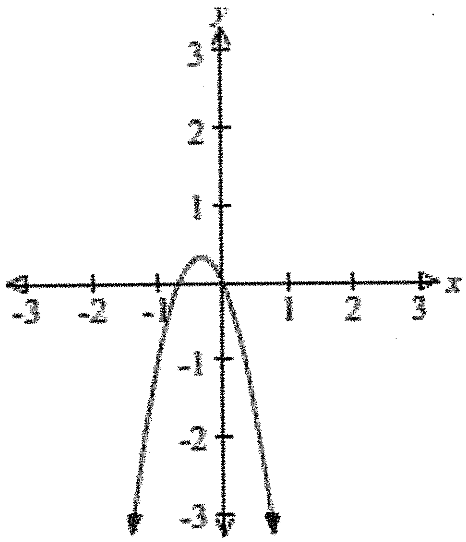
D. $0.5 \times 400 \times \left(\frac{\pi}{5} - \sin\frac{\pi}{5}\right)\ cm$

5. The graph of a function $y = f(x)$ is shown below.

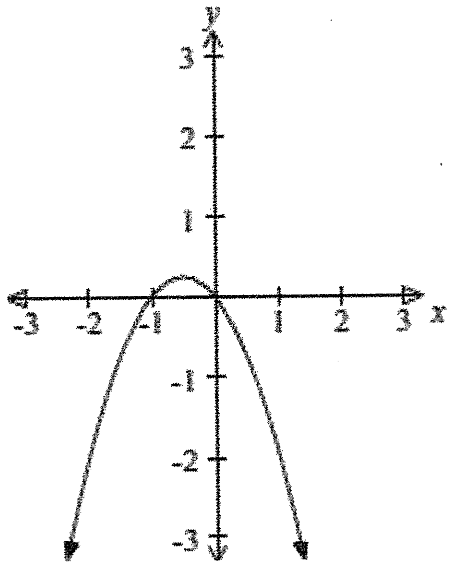


Which graph would represent $y = f'(x)$?

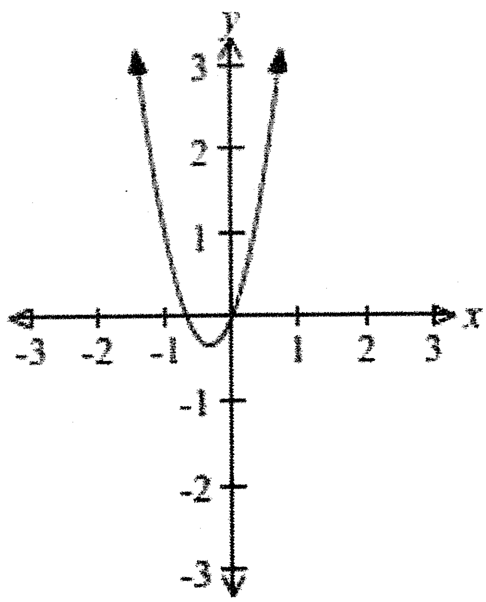
A.



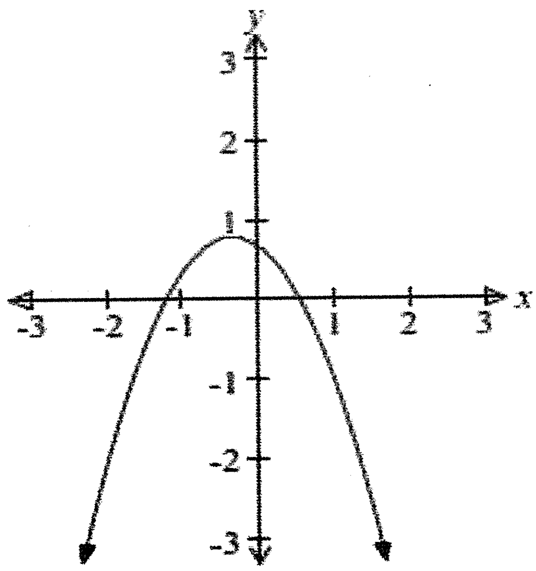
B.



C.



D.



6 The population N ,of a colony of ants grows exponentially according to the formula $N(t) = 550e^{kt}$, where k is the growth constant and t is time in days.

If $N(\ln 4) = 13750$, what is the exact value of k ?

A. $\frac{25}{\ln 4}$

(B.) $\frac{\ln 25}{\ln 4}$

C. $\frac{\ln 4}{25}$

D. $\ln\left(\frac{25}{4}\right)$

$N(\ln 4) = 550e^{k \times \ln 4}$

$13750 = 550e^{k \ln 4}$

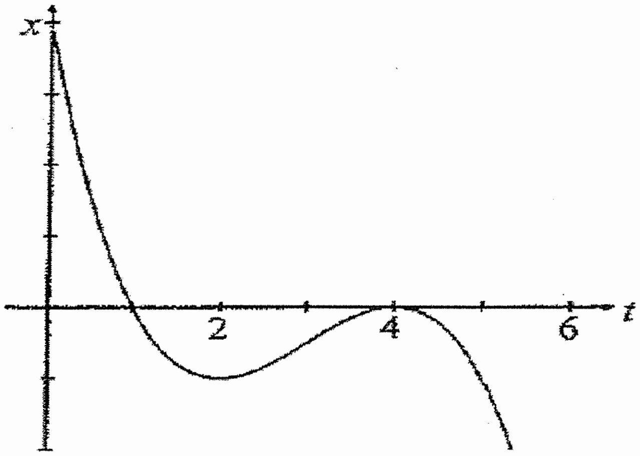
$\frac{13750}{550} = e^{k \ln 4}$

$25 = e^{k \ln 4}$

$\ln 25 = k \ln 4$

$k = \frac{\ln 25}{\ln 4}$

7 The displacement, x metres, from the origin of a particle moving in a straight line at any time, t seconds, is shown in the graph below



When was the particle at rest?

A $t=0$

(B.) $t=2$ and $t=4$

C. $t=1$ and $t=4$

D. $t=1$, $t=2$ and $t=4$

8 What is $\dfrac{d}{d\theta}\left(\dfrac{2-\sin\theta}{\cos\theta}\right)$?

$= \dfrac{\cos\theta(0-\cos\theta)-(2-\sin\theta)(-\sin\theta)}{\cos^2\theta}$

$= \dfrac{-\cos^2\theta+2\sin\theta-\sin^2\theta}{\cos^2\theta}$

$= \dfrac{-(\cos^2\theta+\sin^2\theta)+2\sin\theta}{\cos^2\theta}$

$= \dfrac{-1+2\sin\theta}{\cos^2\theta}$

$= \dfrac{2\sin\theta-1}{\cos^2\theta}$

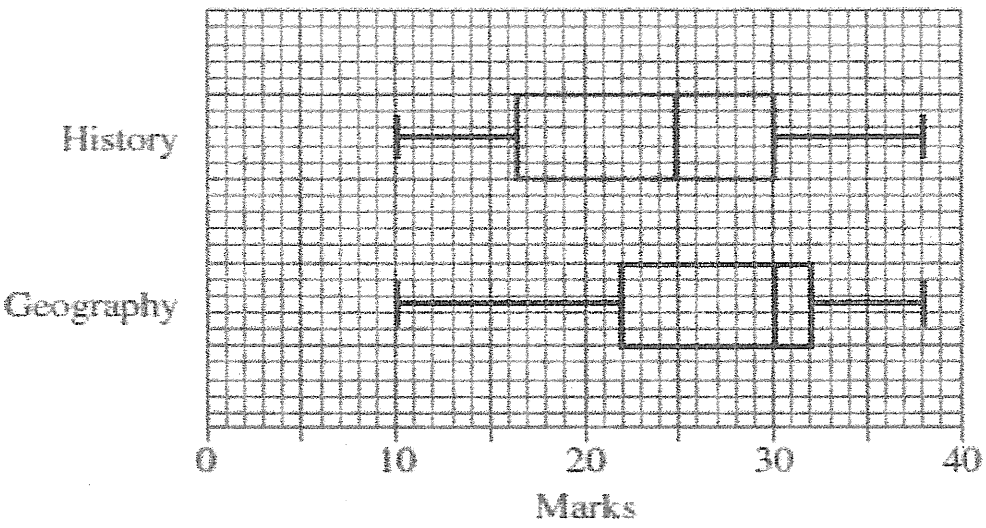
(A.) $\dfrac{2\sin\theta-1}{\cos^2\theta}$

B. $\dfrac{1-2\sin\theta}{\cos^2\theta}$

C. $\dfrac{-\cos^2\theta-2\sin\theta+\sin^2\theta}{\cos^2\theta}$

D. $\dfrac{2\cos\theta+2\sin\theta-1}{\cos^2\theta}$

9 The box and whisker plots given below show the results of a history and a geography test.



In History, 112 students completed the test. The number of students who scored above 30 marks was the same for the History test and the Geography test.

How many students completed the Geography test?

A. 8

B. 50

C. 56

D. 112

In history, $\frac{1}{4} \times 112$ students $= 28$ students got marks above 30.

For Geography, $\frac{1}{2}$ of total number of students got above 30 marks.

$\therefore$ $\frac{1}{2}$ of total students $= 28$

total students $= 28 \times 2 = 56$

10 There are 10 green marbles and W white marbles in a bag. The probability of selecting a white marble is $\frac{4}{9}$. How many more white marbles need to be added to the bag so that the probability of selecting a white marble from the bag is $\frac{3}{5}$?

A. 7

B. 8

C. 14

D. 21

10 G, W white

$P(W) = \frac{W}{10+W} = \frac{4}{9}$

$9W = 40 + 4W$

$5W = 40 \Rightarrow W = \frac{40}{5} = 8$

$\therefore$ 10 G 8 W

Let n numbers of white marbles are added

$\frac{8+n}{18+n} = \frac{3}{5} \Rightarrow 40 + 5n = 54 + 3n$

$2n = 14$

$n = 7$

Section II
Part A

12 marks

Student Number:

Question 11 (2 marks)

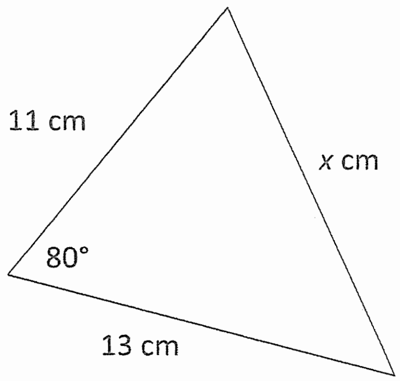
Simplify fully: $\dfrac{3}{x+2} \times \dfrac{x^2+2x}{6x-3}$ 2

$\dfrac{3}{x+2} \times \dfrac{x(x+2)}{3(2x-1)}$ ✓

$= \dfrac{x}{2x-1}$ ✓

Question 12 (2 marks)

In the following diagram, use the Cosine Rule to find the value of x, correct to two decimal places 2



NOT TO SCALE

$x^2 = 11^2 + 13^2 - 2\times 11\times 13 \times \cos 80^\circ$ ✓

$x = \sqrt{11^2 + 13^2 - 2\times 11\times 13 \times \cos 80^\circ}$

≈ 15.50 cm ✓

4

Question 13 (2 marks)

If $f(x)=(x-5)^2$ and $g(x)=\sqrt{x}\ -5$,find $g\left(f(x)\right)$ **2**

$g((x-5)^2)$ ✓

$= \sqrt{(x-5)^2} - 5$

$= |x-5| - 5$ ✓

Question 14 (3 marks)

. The first term of an arithmetic series is 4. The fifth term is four times the third term. Find the sum of the first 10 terms. **3**

$a = 4$ $\qquad t_5 = 4t_3$

$a+4d = 4(a+2d)$ ✓

$a+4d = 4a+8d$

$3a = -4d$

$3\times4 = -4d$

$d = -3$ ✓

$S_{10} = \frac{10}{2}[2a+9d]$

$= 5[2\times4 + 9\times-3]$

$= -95$ ✓

5

Question 15 (3 marks)

Solve $\log_e(2x+1) - \log_e 3 = 3$ 3

$\log_e \frac{2x+1}{3} = 3$ ✓

$\frac{2x+1}{3} = e^3$ ✓

$2x+1 = 3e^3$

$2x = 3e^3 - 1$

$x = \frac{3e^3 - 1}{2}$

$x = \frac{1}{2}(3e^3 - 1)$ ✓

3

PART B

Student Number:

Question 16 (5 marks)

(a) Show that $(\text{cosec}^2\theta - 1)\sin^2\theta = \cos^2\theta$ **2**

L.H.S. $= \left(\dfrac{1}{\sin^2\theta} - 1\right) \times \sin^2\theta$

$= \left(\dfrac{1-\sin^2\theta}{\sin^2\theta}\right) \times \sin^2\theta$ ✓

$= \dfrac{\cos^2\theta}{\sin^2\theta} \times \sin^2\theta$

$= \cos^2\theta = $ R.H.S ✓

(b) Hence, or otherwise solve $(\text{cosec}^2\theta - 1)\sin^2\theta = \dfrac{3}{4}$ for $-\pi \le x \le \pi$ **3**

$\cos^2\theta = \dfrac{3}{4}$

$\cos\theta = \pm\dfrac{\sqrt{3}}{2}$ ✓

$\cos\theta = \dfrac{\sqrt{3}}{2}$ or $\cos\theta = -\dfrac{\sqrt{3}}{2}$

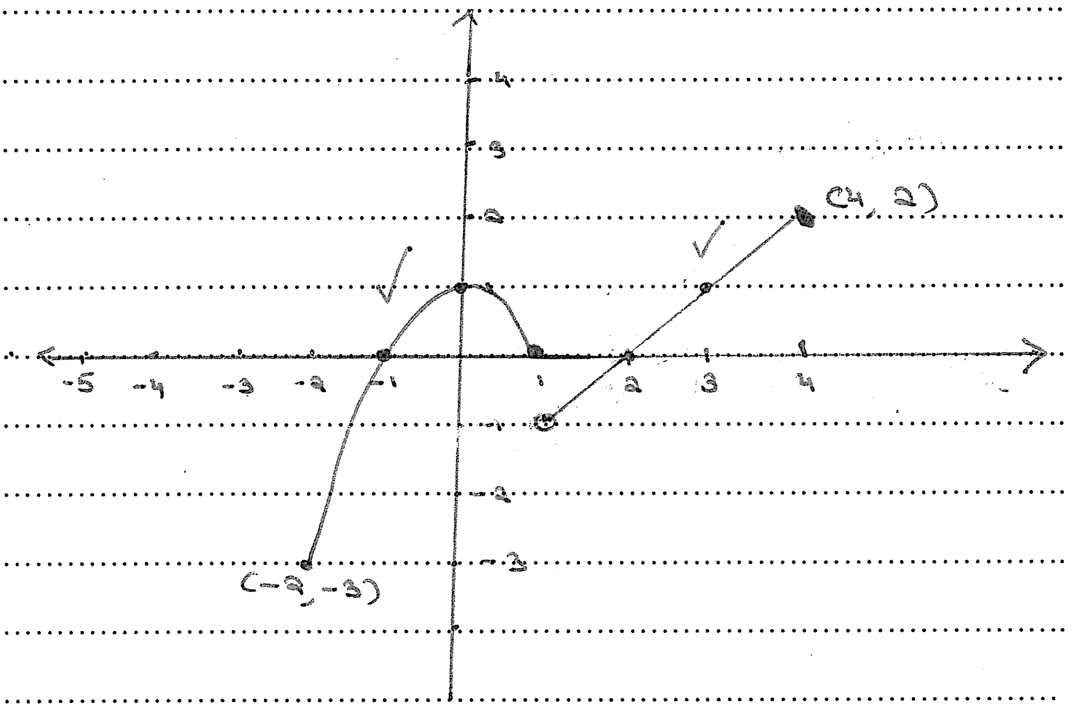
$\theta = -\dfrac{\pi}{6}$ or $\dfrac{\pi}{6}$ ✓ $\theta = -\dfrac{5\pi}{6}$ or $\dfrac{5\pi}{6}$ ✓

5

Question 17 (3marks)

Let $f(x)=\begin{cases}1-x^2 & for \quad x\le 1\\ x-2 & for \quad x>1\end{cases}$

(a) Sketch the function for $-2\le x\le 4$ **2**



(b) What happens to the graph $x=1$? **1**

The graph is discontinuous at $x=1$

3

Question 18 (4 marks)

(a) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sec^2 x \, dx$ giving the answer in exact form with rational denominator. **2**

$= \left[\tan x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$ ✓

$= \tan\frac{\pi}{3} - \tan\frac{\pi}{6}$

$= \sqrt{3} - \frac{1}{\sqrt{3}}$

$= \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ ✓

(b) Find $\int x^2 \left(x^3 - 1\right)^5 \, dx$ **2**

$= \frac{1}{3} \int 3x^2 (x^3 - 1)^5 \, dx$ ✓

$= \frac{1}{3} \times \frac{(x^3 - 1)^6}{6} + C$ ✓

$= \frac{(x^3 - 1)^6}{18} + C$

4

Question 19 (6 marks)

A function is given by $f(x) = 3 + 4x^3 - x^4$. **3**

(a) Find the stationary points and determine their nature.

$f'(x) = 12x^2 - 4x^3$

$f''(x) = 24x - 12x^2$

$f'(x) = 0 \Rightarrow 12x^2 - 4x^3 = 0$

$4x^2(3 - x) = 0$

$x = 0$ or $x = 3$ ✓

At $x = 3$ $f''(x) = 24 \times 3 - 12 \times 3^2 = -36 < 0$

∴ The curve has max at $x = 3$

when $x = 3$ $y = 3 + 4 \times 3^3 - 3^4 = 30$

∴ $(3, 30)$ is point of maximum ✓

when $x = 0$ $f''(x) = 0$

x	-1	0	1
$f''(x)$	-36 $-$ive	0	12 $+$ive

∴ At $x = 0$, the curve has horizontal point of inflexion

when $x = 0$, $y = 3$

∴ $(0, 3)$ is a horizontal point of inflexion ✓

3

Question 19 (continues)

(b) Show that there is a point of inflexion at (2,19) on the curve. **1**

$f''(x) = 0 \Rightarrow 24x - 12x^2 = 0$

$12x(2 - x) = 0$

$\Rightarrow x = 0$ or $x = 2$

When $x = 0$, the curve has horizontal point of inflexion (proved in part a)

When $x = 2$

x	1	2	3
$f''(x)$	12 +ive	0	−36 −ive

∴ at $x = 2$, the curve has a point of inflexion ✓

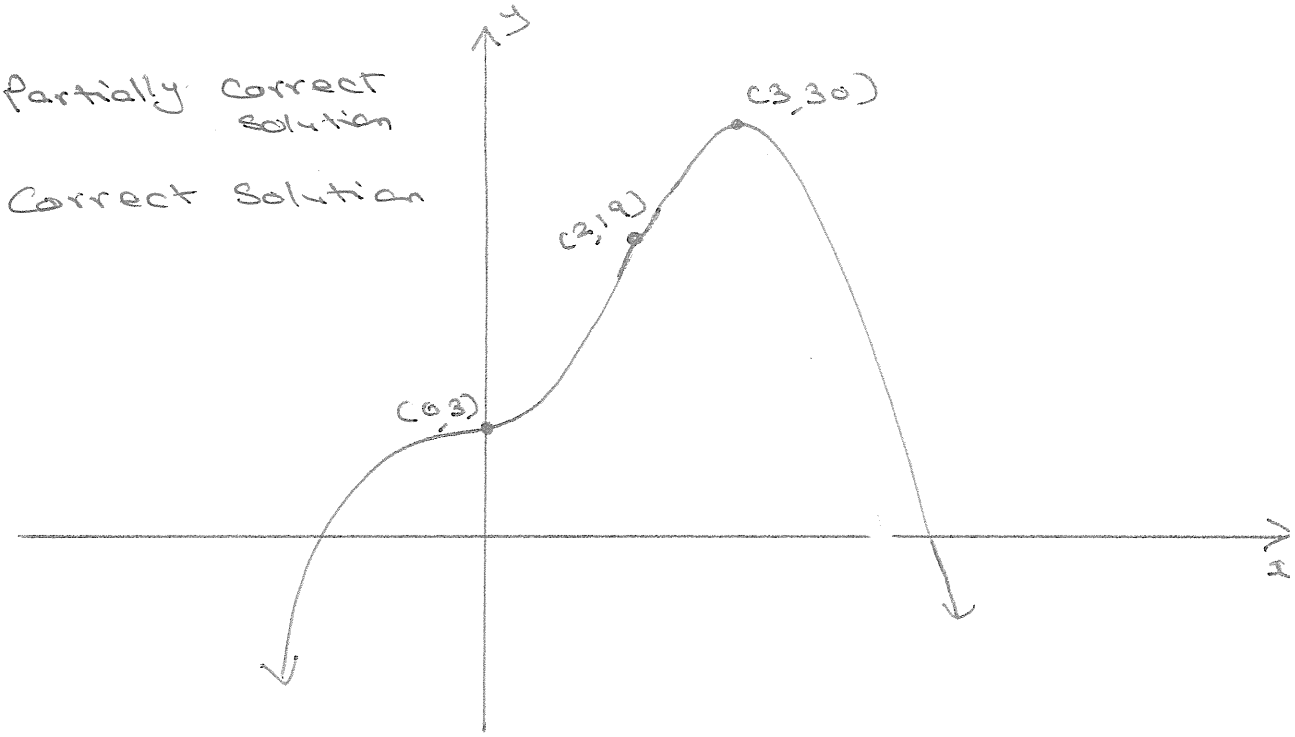
When $x = 2$, $y = 3 + 4 \times 2^3 - 2^4 = 19$

∴ The curve has a point of inflexion at (2, 19)

(c) Hence, Sketch the graph of the curve and clearly label the stationary points and the points of inflexion on the curve. DO NOT determine the x intercepts of the curve. **2**

✓ Partially correct solution

✓✓ Correct Solution



PART C — 15 marks

Student Number

Question 20 (5 marks)

(a) Show that $\dfrac{1+e^x}{1-e^x} = 1+\dfrac{2e^x}{1-e^x}$ 2

$LHS = \dfrac{1+e^x}{1-e^x}$

$= \dfrac{1-e^x+2e^x}{1-e^x}$ ✓

$= \dfrac{1-e^x}{1-e^x} + \dfrac{2e^x}{1-e^x}$

$= 1 + \dfrac{2e^x}{1-e^x}$ ✓

$= RHS$

Alternative soln:

$RHS = 1 + \dfrac{2e^x}{1-e^x}$

$= \dfrac{1-e^x+2e^x}{1-e^x}$ ✓

$= \dfrac{1+e^x}{1-e^x}$ ✓

$= LHS$

(b) Hence, find $\displaystyle\int \frac{1+e^x}{1-e^x}\,dx$ 3

$\displaystyle\int \frac{1+e^x}{1-e^x}\,dx = \int \left(1 + \frac{2e^x}{1-e^x}\right) dx$ ✓

$\displaystyle= \int \left(1 + (-2)\left(\frac{-e^x}{1-e^x}\right)\right) dx$ ✓

$= x - 2\ln|1-e^x| + C$ ✓

5

Question 21 (3 marks)

A normal to the graph of $y = 2\sqrt{x}$ has the equation $y = -3x + b$. Find the value of b. **3**

$y' = 2 \times \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}}$

gradient of normal $= -\sqrt{x}$ ✓

$-\sqrt{x} = -3 \Rightarrow x = 9$

when $x = 9$ $y = 2 \times \sqrt{9} = 2 \times 3 = 6$

∴ The point on the curve is $(9, 6)$ ✓

$y = -3x + b$

$6 = -3 \times 9 + b$

$b = 33$ ✓

3

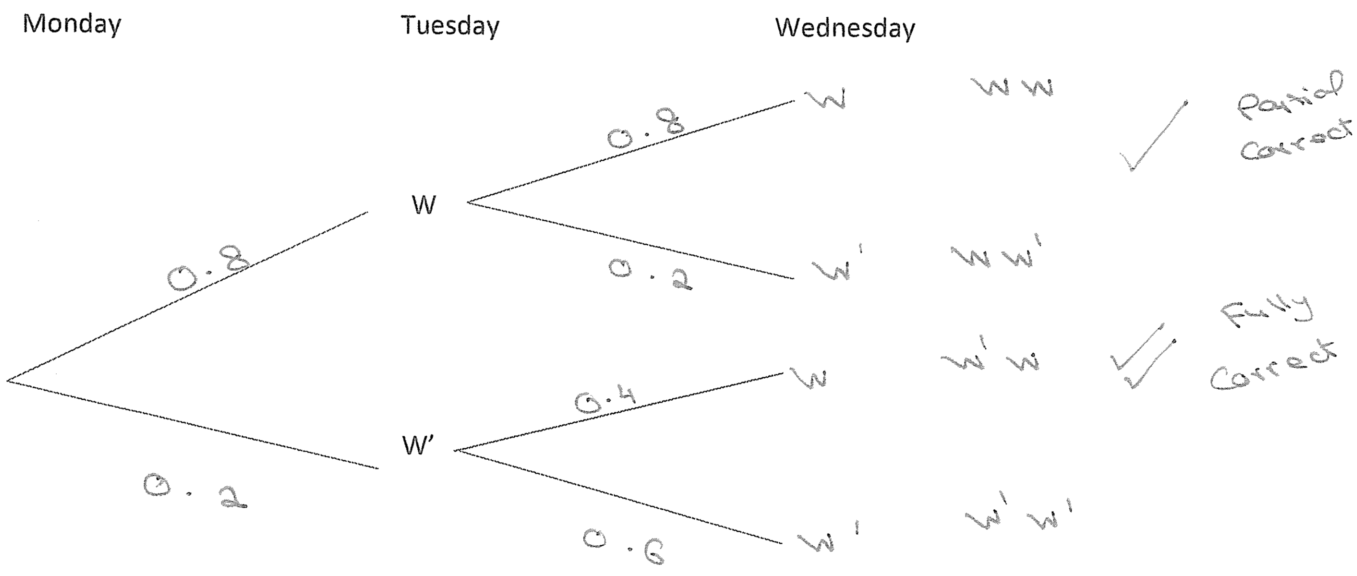
Question 22 (4 marks)

The probability of Sirhind soccer team winning their next game is dependent on whether or not they win the previous game.

If they won the previous game, the probability of winning their next game is 0.8
If they did not win their previous game, then the probability of winning the next game is 0.4

They won on Monday and then they played on Tuesday and Wednesday.

(a) Represent the above information using the tree diagram given below 2



(b) Find the probability that the Sirhind soccer team do not win on Wednesday. 2

P(WW' or W'W')

= P(WW') + P(W'W')

= 0.8×0.2 + 0.2×0.6 = 0.16 + 0.12 = 0.28

4

Question 23 (3 marks)

For the probability distribution below, find the values of m and n given that the expected value of the distribution is 4.8 **3**

x	0	2	3	7	n
$P(X=x)$	0.1	0.3	m	0.3	0.2

$\sum p(x) = 1$

$0.1 + 0.3 + m + 0.3 + 0.2 = 1$

$m + 0.9 = 1$

$m = 0.1$ ✓

$\sum x\,p(x) = 4.8$

$0.1 \times 0 + 0.3 \times 2 + m \times 3 + 0.3 \times 7 + 0.2 \times n = 4.8$ ✓

$0.6 + 3m + 2.1 + 0.2n = 4.8$

$2.7 + 3 \times 0.1 + 0.2n = 4.8$

$2.7 + 0.3 + 0.2n = 4.8$

$0.2n = 1.8$

$n = 9$ ✓

3

PART D — 17 marks

Student Number

Question 24 (5 marks)

If $f(x)=\log_e\left(x+\sqrt{x}\right)$

(a) Show that $f'(x)=\dfrac{2\sqrt{x}+1}{2x\left(\sqrt{x}+1\right)}$ 3

$f(x) = \log_e(x+\sqrt{x})$

$f'(x) = \dfrac{1}{x+\sqrt{x}}\left[1+\dfrac{1}{2\sqrt{x}}\right]$ ✓ ✓

$= \dfrac{1}{x+\sqrt{x}} \times \dfrac{2\sqrt{x}+1}{2\sqrt{x}}$

$= \dfrac{2\sqrt{x}+1}{(x+\sqrt{x})\times 2\sqrt{x}}$

$= \dfrac{2\sqrt{x}+1}{\sqrt{x}(\sqrt{x}+1)\times 2\sqrt{x}}$

$= \dfrac{2\sqrt{x}+1}{2x(\sqrt{x}+1)}$ ✓

3

Question 24 (continued)

(b) Hence, when $x=2$, Show that $f'(x)=\dfrac{2\sqrt{x}+1}{2x\left(\sqrt{x}+1\right)}$ can be written in the form $a+b\sqrt{2}$ **2**

$$f'(2) = \frac{2\sqrt{2}+1}{2\times 2(\sqrt{2}+1)}$$

$$= \frac{2\sqrt{2}+1}{4(\sqrt{2}+1)}$$

$$= \frac{2\sqrt{2}+1}{4(\sqrt{2}+1)} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$ ✓

$$= \frac{(2\sqrt{2}+1)(\sqrt{2}-1)}{4(\sqrt{2}+1)(\sqrt{2}-1)}$$

$$= \frac{4-2\sqrt{2}+\sqrt{2}-1}{4(2-1)}$$

$$= \frac{3-\sqrt{2}}{4}$$

$$= \frac{3}{4} - \frac{1}{4}\sqrt{2}$$ ✓

2

Question 25 (9 marks)

Consider the function defined by $f(x) = e^{2x}(1-x)$ where $-3 \le x \le 1$

(a) Complete the following table **1**

x	-3	-2	-1	0	1
$f(x)$	0.01	0.05	0.27	1	0

✓

(b) Using the trapezoidal rule with five function values, approximate the area under the curve $f(x)$ for $-3 \le x \le 1$ **2**

$A \doteqdot \frac{1-(-3)}{2\times 4}\left[f(-3)+f(1)+2\left(f(-2)+f(-1)+f(0)\right)\right]$ ✓

$= \frac{4}{8}\left[0.01+0+2\left(0.05+0.27+1\right)\right]$

$= \frac{1}{2} \times 2.65$

$= 1.325$ units^2 ✓

3

Question 25 (continued)

(c) Differentiate $f(x) = e^{2x}(1-x)$ and show that the function has only one stationary point. **3**

$f'(x) = e^{2x} \times -1 + (1-x) \times 2e^{2x}$

$= -e^{2x} + 2e^{2x} - 2xe^{2x}$

$= e^{2x} - 2xe^{2x}$

$= e^{2x}(1-2x)$ ✓

$f'(x) = 0$

$e^{2x}(1-2x) = 0$

$1-2x = 0$ ($\because e^{2x}$ is always > 0) ✓

$\therefore x = \frac{1}{2}$ ✓

When $x = \frac{1}{2}$, $f(\frac{1}{2}) = e^{2\times\frac{1}{2}}(1-\frac{1}{2})$

$= e(\frac{1}{2}) = \frac{e}{2}$

$\therefore$ The curve has only one stationary point $(\frac{1}{2}, \frac{e}{2})$

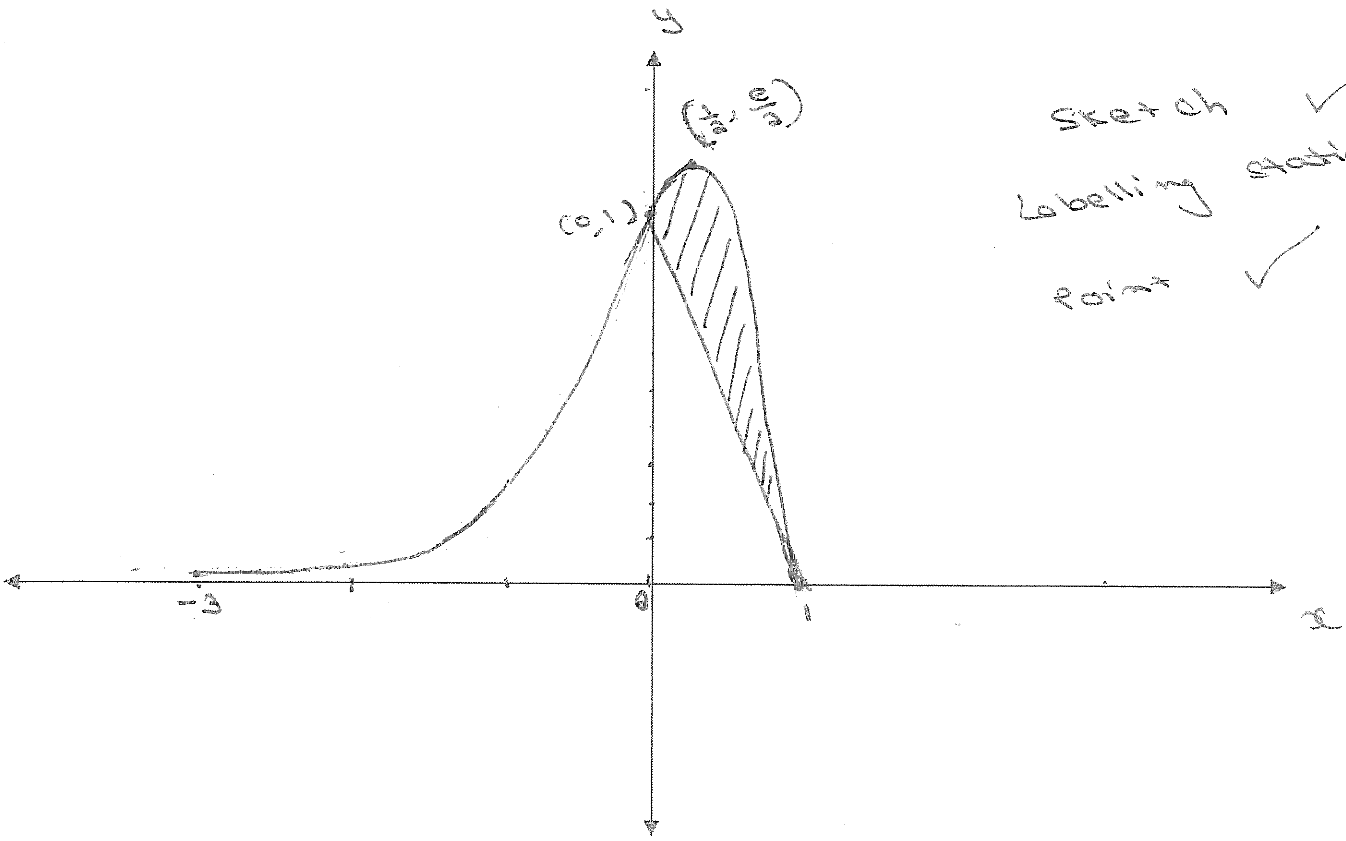
or $(\frac{1}{2}, 1.36)$

3

Question 25 (continued)

(d) Without further calculus, use the table of values from part (a) to sketch

$f(x) = e^{2x}(1-x)$ for $-3 \le x \le 1$, and show the stationary point on the curve. **2**



Sketch ✓
Labelling stationary point ✓

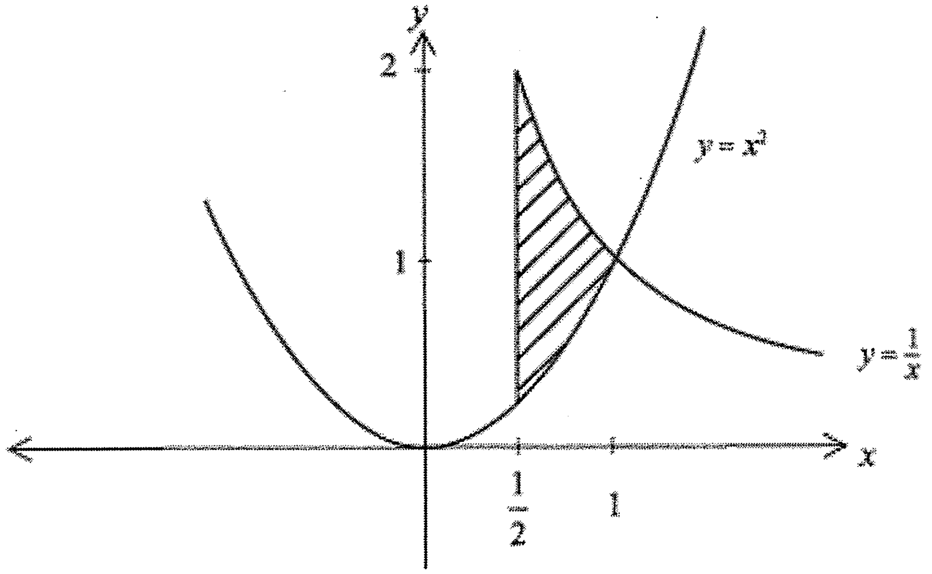
(e) From the diagram, decide whether the approximation underestimates or overestimates the true value of the area under the curve. Give a brief reason. **1**

Underestimates, because between 0 and 1 the straight line which enclosed its area to be calculated for this interval is significantly below the diagram. ✓

3

Question 26 (3 marks)

Calculate the exact area of the shaded region in the diagram below enclosed by the curves $y = x^2$, $y = \frac{1}{x}$ and the line $x = \frac{1}{2}$ **3**



Required Area

$= \int_{\frac{1}{2}}^{1} \left(\frac{1}{x} - x^2\right) dx$ ✓

$= \left[\ln x - \frac{x^3}{3}\right]_{\frac{1}{2}}^{1}$ ✓

$= \left(\ln 1 - \frac{1}{3}\right) - \left(\ln \frac{1}{2} - \frac{1}{3} \times \frac{1}{8}\right)$

$= 0 - \frac{1}{3} - \ln \frac{1}{2} + \frac{1}{24}$

$= \frac{-7}{24} - \ln \frac{1}{2}$ or $-\frac{7}{24} + \ln 2$ ✓

3

PART E —— 18 marks

Student Number

Question 27 (3 marks)

Consider the function $f(x)=\frac{1}{2}-\frac{1}{2^x+1}$

(a) Show that $\frac{1}{2}-\frac{1}{2^x+1}=\frac{2^x-1}{2(2^x+1)}$ 1

L.H.S. $=\frac{1}{2}-\frac{1}{2^x+1}$

$=\frac{2^x+1-2}{2(2^x+1)}$

$=\frac{2^x-1}{2(2^x+1)}$ ✓

(b) Hence determine whether $f(x)$ is even, odd or neither. Show all working. 2

$f(-x)=\frac{2^{-x}-1}{2(2^{-x}+1)}$

$=\frac{\frac{1}{2^x}-1}{2(\frac{1}{2^x}+1)}$

$=\frac{1-2^x}{2(1+2^x)}$

$=-\frac{(2^x-1)}{2(2^x+1)}$

$=-f(x)$

Alternative Soln :

$f(-x)=\frac{2^{-x}-1}{2(2^{-x}+1)}\times\frac{2^x}{2^x}$

$=\frac{1-2^x}{2(1+2^x)}$

$=\frac{-(2^x-1)}{2(2^x+1}$

$=-f(x)$

$\therefore f(x)$ is an odd fn.

3

Question 28 (6 marks)

The velocity v of a particle initially at the origin is given by $v(t) = \sin\left(\frac{1}{4}t\right)$, in metres per second.

(a) Find the displacement function $x(t)$ **2**

$x(t) = \int v(t)\,dt = \int \sin\left(\frac{1}{4}t\right)dt$

$= -4\cos\frac{1}{4}t + C$ ✓

When $t=0$, $x=0 \Rightarrow 0 = -4\cos\frac{1}{4}\times 0 + C$

$0 = -4\cos 0 + C \Rightarrow 0 = -4 + C \Rightarrow C = 4$

$\therefore x(t) = -4\cos\frac{1}{4}t + 4$ ✓

(b) Find the acceleration function $a(t)$ 1

$a(t) = \frac{d}{dt}v(t) = \frac{d}{dt}\left(\sin\frac{1}{4}t\right)$

$a(t) = \frac{1}{4}\cos\left(\frac{1}{4}t\right)$ ✓

(c) Find the value of x when $t = 4\pi$ 1

When $t = 4\pi$

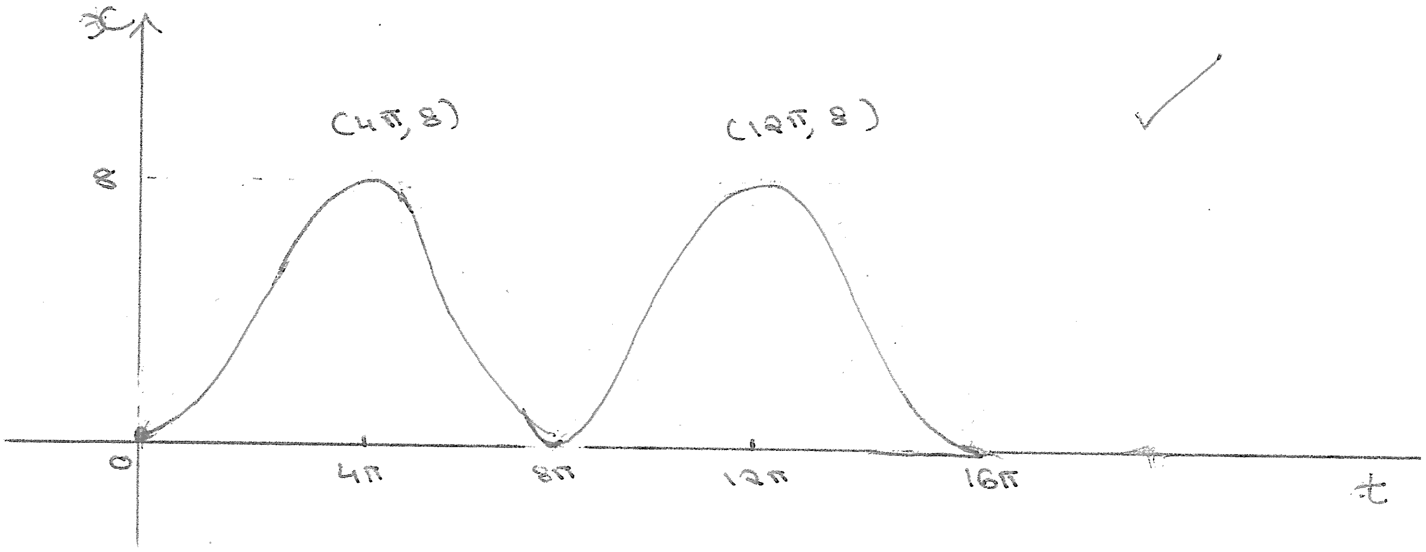
$x(t) = -4\cos\frac{1}{4}\times 4\pi + 4$

$= -4\cos\pi + 4 = -4\times -1 + 4 = 8$ m ✓

4

Question 28 (continued)

(d) Sketch the displacement-time graph on the domain $0 \le t \le 16\pi$ and briefly describe the motion. **2**



The particle oscillates between x = 0 to x = 8 with period 8 π seconds ✓

1 - correct shape, amplitude marking on the *y*-axis, all labels on *x*-axis.
1 - Describe motion including, at the least, amplitude or period.

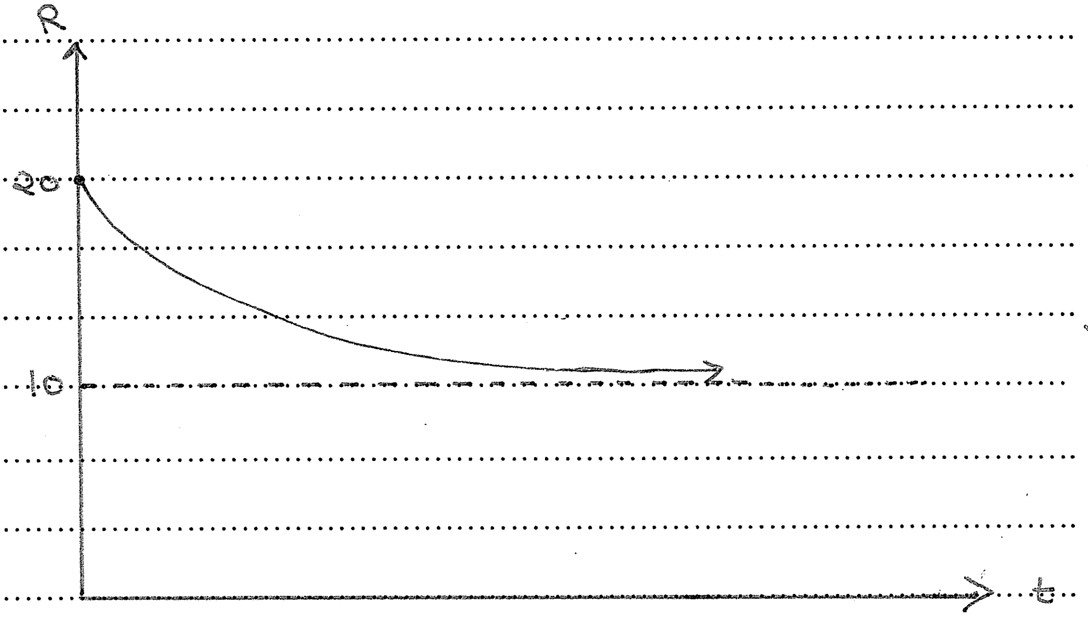
2

Question 29 (7 marks)

The rate of fuel burn, t minutes after the engine starts operation, R kg per minute, is given by the relation

$$R = 10 + \frac{10}{1+2t}$$

(a) Draw a sketch of R as a function of t , $t \geq 0$ **2**



✓ Partially correct

✓✓ Correct answer

(b) What is the rate of burn, R, after 7 minutes? **1**

when $t = 7$

$R = 10 + \frac{10}{1+2\times7} = 10 + \frac{10}{15}$

$= 10\frac{2}{3}$ kg / minute ✓

3

Question 29 (continued)

(c) What value does R approaches as t becomes very large? 1

As $t \to \infty$

$\frac{10}{1+2t} \to 0$

$R \to 10$ ✓

(d) Calculate the total amount of the fuel burnt in the first 7 minutes. 3

Total amount of the fuel burnt in first 7 minutes $= \int_0^7 R\, dt$ ✓

$= \int_0^7 \left(10 + \frac{10}{1+2t}\right) dt$

$= \left[10t + 10 \times \frac{\ln|1+2t|}{2}\right]_0^7$ ✓

$= 5\left[2t + \ln|1+2t|\right]_0^7$

$= 5[2\times7 + \ln|1+2\times7| - (2\times0 + \ln 1)]$

$= 5[14 + \ln 15]$ kg ✓

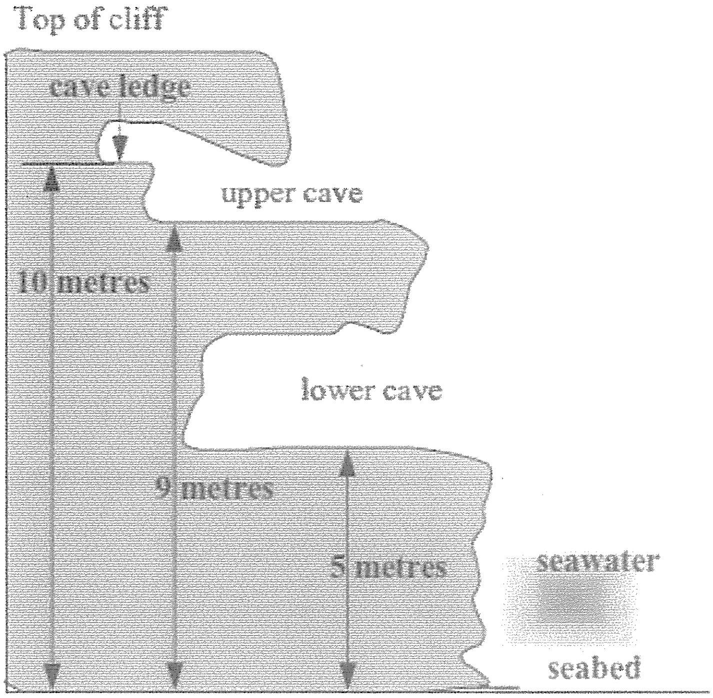
4

PART F

Student Number

Question 30 (5 marks)

Carol, Mary and Manar arrive at the top of a cliff at 9:00 am on a particular Saturday morning. They are planning to explore the lower cave in the cliff.



The height, h metres of the seawater is given by the function $h(t) = 7 - 4\sin\left(\frac{\pi}{7}t\right)$, where t is the number of hours after 9:00 am, on Saturday morning.

(a) What is the height of the water when the girls first arrive at the top of the cliff? 1

when $t = 0$, $h(t) = 7 - 4\sin\left(\frac{\pi}{7} \times 0\right) = 7 - 0$

$= 7$ m

(b) The girls notice that the height of the water is falling and they wait for the water to fall so that they can explore the lower cave. When the water is 4 metres and falling, they enter the lower cave.

What time do the girls enter the lower cave? 2

$7 - 4\sin\left(\frac{\pi}{7}t\right) = 4$

$-4\sin\left(\frac{\pi}{7}t\right) = -3$

$\sin\left(\frac{\pi}{7}t\right) = \frac{3}{4}$

$\frac{\pi}{7}t = \sin^{-1}\frac{3}{4}$

$\Rightarrow t = \frac{7}{\pi}\sin^{-1}\frac{3}{4}$

$\doteqdot$ 1hr 53min

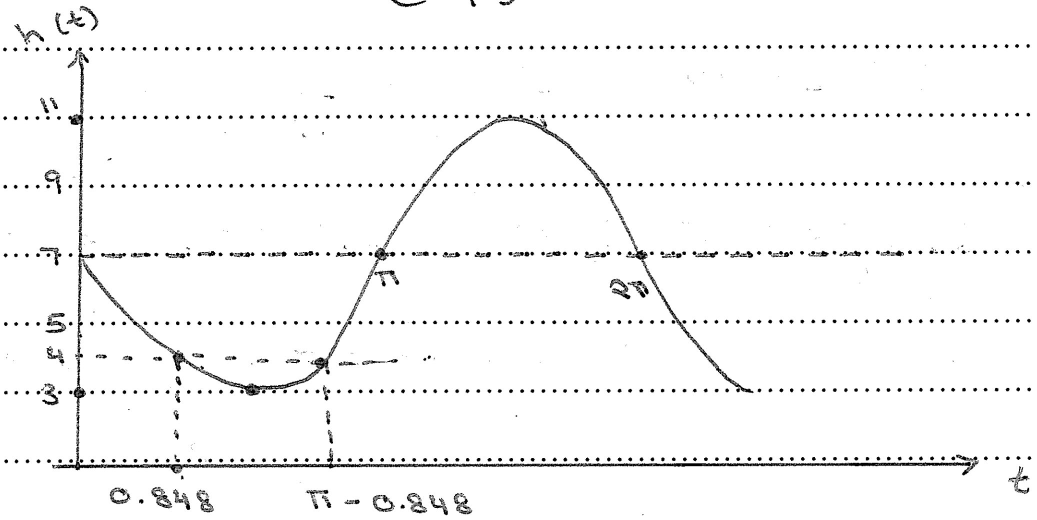
∴ the girls enter the lower cave at 10:53am.

Q30 (continued)

(c) The girls plan to leave the lower cave when the water level is again at 4 metres and rising.

Calculate how long will they have to explore the lower cave to the nearest minute? **2**

$7 - 4\sin\left(\frac{\pi}{7}t\right) = 4$



$\sin\frac{\pi}{7}t = \frac{3}{4}$

$\frac{\pi}{7}t = \sin^{-1}\frac{3}{4}$

$\frac{\pi}{7}t = 0.84810, \quad \pi - 0.8481$

$t = 1.8897, \quad 5.1103$

difference $= 5.1103 - 1.8897 = 3.2206$

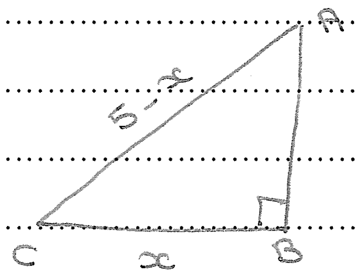
$\approx$ 3 hours 13 minute

The girls will have approx. 3 hours and 13 minutes to explore the lower cave.

Question 31 (7 marks)

A wire of length 5 metres is to be bent to form the hypotenuse and base of a right angled triangle ABC with right angle at B. Let the length of the base BC be x ~~metres~~. metres

(a) Write an expression for the length of the hypotenuse AC in terms of x. **1**



$AC = 5 - x$ ✓

(b) Show that the area of the triangle ABC is $\frac{1}{2}x\sqrt{25-10x}$ square ~~meters~~ metres. **2**

$AB^2 = (5-x)^2 - x^2$

$= 25 + x^2 - 10x - x^2 = 25 - 10x$

$AB = \sqrt{25-10x}$

Area of triangle $ABC = \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times x \times \sqrt{25-10x}$

$= \frac{1}{2} x \sqrt{25-10x}$ sq. metres

Question 31(continued)

(c) Find the maximum possible area of the triangle ABC. **4**

$A = \frac{1}{2} x \sqrt{25-10x}$

$\frac{dA}{dx} = \frac{1}{2}\left[1 \times \sqrt{25-10x} + x \times \frac{1}{2\sqrt{25-10x}} \times -10\right]$

$= \frac{1}{2}\left[\frac{25-10x-5x}{\sqrt{25-10x}}\right] = \frac{1}{2}\frac{25-15x}{\sqrt{25-10x}}$ ✓

$\frac{dA}{dx} = 0 \Rightarrow \frac{1}{2}\frac{25-15x}{\sqrt{25-10x}} = 0$

$\Rightarrow 25 - 15x = 0 \Rightarrow x = \frac{25}{15} = \frac{5}{3}$ ✓

≈ 1.67

x	1	$\frac{5}{3}$	2
$\frac{dA}{dx}$	1.29	0	−1.12

+ive −ive

$\frac{dA}{dx}$ changes the sign from +ive to −ive

$\therefore A$ is max when $x = \frac{5}{3}$ ✓

maximum area $= \frac{1}{2} \times \frac{5}{3} \times \sqrt{25 - \frac{10 \times 5}{3}}$

$= \frac{5}{6}\sqrt{\frac{25}{3}}$

$= \frac{25}{6\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{25\sqrt{3}}{18}$ Sq. metres ✓

4

End of Paper

2022 Trial Higher School Certificate Examination

Mathematics Advanced

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	2 + 4 =	(A) 2	(B) 6	(C) 8	(D) 9
		A ○	B ●	C ○	D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ●	B ⊗	C ○	D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗	B ⊗ (correct)	C ○	D ○

1.	A ○	B ○	C ●	D ○
2.	A ○	B ○	C ○	D ●
3.	A ○	B ○	C ○	D ●
4.	A ●	B ○	C ○	D ○
5.	A ●	B ○	C ○	D ○
6.	A ○	B ●	C ○	D ○
7.	A ○	B ●	C ○	D ○
8.	A ●	B ○	C ○	D ○
9.	A ○	B ○	C ●	D ○
10.	A ●	B ○	C ○	D ○